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Spatial Frequency Domain Error Budget

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SPATIAL FREQUENCY DOMAIN ERROR BUDGET

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1. Abstract

The aim of this paper is to describe a methodology for designing and characterizing machines used to manufacture or inspect parts with spatial-frequency-based specifications. At Lawrence Livermore National Laboratory, one of our responsibilities is to design or select the appropriate machine tools to produce advanced optical and weapons systems. Recently, many of the component tolerances for these systems have been specified in terms of the spatial frequency content of residual errors on the surface. We typically use an error budget as a sensitivity analysis tool to ensure that the parts manufactured by a machine will meet the specified component tolerances. Error budgets provide the formalism whereby we account for all sources of uncertainty in a process, and sum them to arrive at a net prediction of how "precisely" a manufactured component can meet a target specification. Using the error budget, we are able to minimize risk during initial stages by ensuring that the machine will produce components that meet specifications before the machine is actually built or purchased. However, the current error budgeting procedure provides no formal mechanism for designing machines that can produce parts with spatial-frequency-based specifications. The output from the current error budgeting procedure is a single number estimating the net worst case or RMS error on the work piece. This procedure has limited ability to differentiate between low spatial frequency form errors versus high frequency surface finish errors. Therefore the current error budgeting procedure can lead us to reject a machine that is adequate or accept a machine that is inadequate. This paper will describe a new error budgeting methodology to aid in the design and characterization of machines used to manufacture or inspect parts with spatial-frequency-based specifications. The output from this new procedure is the continuous spatial frequency content of errors that result on a machined part. If the machine does not meet specifications, the procedure identifies the sources of the critical errors. We would then evaluate these errors and either reduce the errors through design improvements or modifications to cutting parameters (spindle speed, feed, etc.) or select a different candidate machine if improvements were not practical.

2. Background

The principles of designing precision instruments for meeting challenging tolerance requirements have a rich history [1]. Likewise, the methodologies for analyzing the errors in experimental data and performing differential sensitivity analyses are well-documented [2,3]. Yet the first clear formalization of error budgeting applied to precision engineering appears to originate in the analysis by R. Donaldson during the design of the Large Optics Diamond Turning Machine at Lawrence Livermore National Laboratory (LLNL) [4]. Donaldson's formalism is referenced in current textbooks [5] and is the basis for subsequent machine designs at LLNL [6].

Figure 1 shows flowcharts for both the conventional and the new error budget procedure and how they differ. The upper portion of Figure 1 shows Donaldson's flowchart that illustrates the mapping of error sources onto a work piece geometry. The first step of the conventional error budget is to identify the physical influences that generate the dimensional errors that propagate through the machine tool. These include effects such as thermal gradients and temperature variability, bearing noise, fluid turbulence in cooling passages, way non-straightness, etc. The next step is to determine how this source couples to the machine. A coupling mechanism converts these physical influences into a displacement that has a direct influence on machine performance. An example of a coupling mechanism is the thermal expansion that may transform a time-varying heat source in the vicinity of the machine into a machine way distortion. These displacements represent dimensional changes in the system. A single peak-to-valley number is usually used to quantify the dimensional changes, not differentiating between the spatial frequency content of the error. The next step is to sum all the contributing errors using an appropriate combinatorial algorithm. Literature suggests a variety of combinatorial algorithms [7]. The last step in the error budgeting procedure is to transform these errors into the work piece coordinate system. To convert these machine displacements into the errors that would reside on the work piece surface in the directions of interest, we must consider the tool path (i.e. feed rates, spindle speeds, etc.). The output from this procedure is a single number predicting the net error that would result on a machined work piece. We would then compare this number to the part specifications. If the prediction meets target specifications, we would accept the machine design under evaluation. If the prediction does not meet specifications, we would evaluate methods to improve this design by observing which

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sources are the dominating contributing errors. In this way we can evaluate the cost versus accuracy of different candidate designs. If improvements can be made to an existing design, we would make those changes to the error budget and reevaluate the net error. If the modifications are not practical, we would then consider an entirely new design, or possibly reevaluate the specifications.

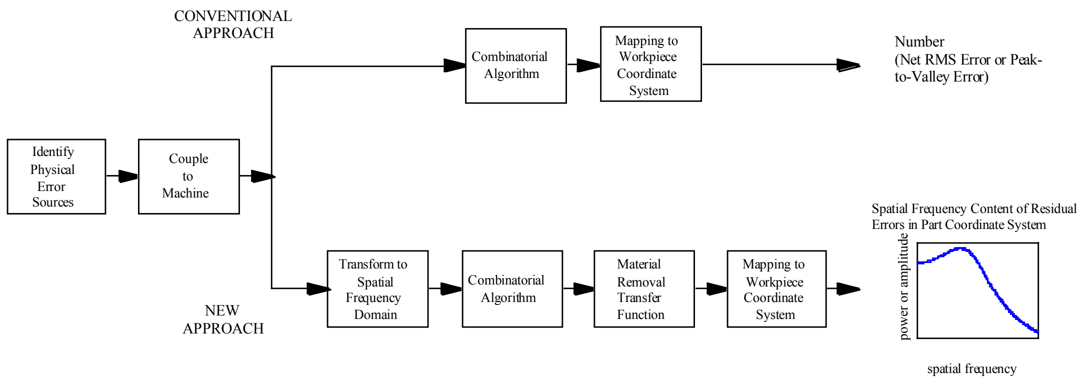

Figure 1. Flowchart comparison between conventional approach and new approach.

3. Technical Approach

The lower portion of Figure 1 shows the new error budget approach. The first two steps, identifying the sources and how they couple to the machine, are identical. However, the next step in the new approach converts the elemental errors into the frequency domain. The next step is to combine the errors in the frequency domain. The combinatorial rule is a completely new algorithm with a statistical foundation. The next step considers the machining process. The conventional approach assumes that the machine and cutting process don't damp or amplify the error sources, while the proposed approach considers the dynamics of the cutting process. The last step is to transform the errors into the part coordinate system. The output from this process is the continuous spectrum of errors that would result at all spatial frequencies on the part.

3.1 Combinatorial Rule

We have developed a combinatorial rule for the addition of the frequency content of each elemental error. The key to the combinatorial algorithm is to consider the spectrum of each elemental error as the sum of sinusoidal errors at specific frequencies. The addition of two sinusoidal signals at a given frequency results in a sinusoidal signal with the same frequency, but the amplitude can vary anywhere from the direct difference to the sum of the two amplitudes depending on the phase shift between the two signals. We first identify all elemental errors that are correlated and appropriately sum the amplitudes of these errors. We then consider the phase shift between the remaining elemental errors to be uniformly distributed variables between 0 and 2π. We have analytically shown that the expected value of the square of the net amplitude is equal to the sum of the squares of the amplitudes of each elemental error. This is equivalent to saying that the expected net power spectral density (PSD) is the sum of the elemental PSDs. Furthermore, we can now determine the probability distribution function of the net error with the use of a Monte Carlo simulation. The 95% confidence limit of the net PSD is approximately three times the mean, and the 99% confidence limit is approximately 4.6 times the mean. This is significantly less than the worst case error. For example, if 25 errors of equal amplitude were summed, the worst case net PSD would be over eight times larger than the 95% confidence limit, and over five times larger than the 99% confidence limit.

3.2 Cutting Process Transfer Function

The purpose of the cutting process transfer function is to convert the motion of the tool in free space to the motion of the tool in the part during the cutting process. This step is necessary because current error characterization procedures measure the error motion of the tool in an open loop sense. The loop is closed when tool is in contact with the part during the cutting process. Differences occur when the loop is closed due to static and dynamic stiffness of the machining system. The conventional error budgeting procedure assumed that the measured motion of the tool in free space is the same as the motion of the tool in the part during cutting, or in other words it assumes that the transfer function equals one. This may be true for a specific frequency band, although we assume that the cutting process will act as a filter with the ability to amplify or suppress the input error motion. For example, if the frequency content of one error were close to the machine resonance, we would expect this error to be amplified. In general, however, during precision machining we (1) design machines with high first modes, and (2) avoid machining conditions that are at the machine resonances. For precision operations, we expect that the machining operation will appear as a low-pass filter, where the low frequency gain is one and the high frequency gain decreases.

3.3 Mapping the Errors into the Work piece Coordinate System

Given the frequency content of the error motion of the tool during cutting, we must take into consideration the path of the tool and the tool geometry determine the frequency content of the residual surface errors on the work piece. Typical tools with a round cutting edge impart a nominal surface finish, or scalloping, during turning, even for a process with no errors. Next, we consider the exact path of the tool during the entire cutting procedure to map these errors onto the relevant work piece coordinate system.

For example, during a facing operation on a diamond turning machine, the part turns while the tool remains stationary. Consider the spatial frequency content of a radial trace across the work piece. The turning process can be considered a sampling mechanism. The radial trace is composed of the time domain sampling of the tool motion once every revolution of the part. Once every revolution, the tool falls on the radial trace of interest, leaving behind the signature of the tool as well as any error motions.

The description of the process so far has been in the time domain. However, we are interested in the frequency domain. Sampling in the time domain can be decomposed into a multiplication procedure of the original time domain signal by a series of impulses. Since multiplication in the frequency domain is equivalent to convolution in the frequency domain, the sampling procedure is converted to the frequency domain by a convolution process. Note that unavoidable aliasing occurs for errors with higher frequency content than the rotational speed of the spindle. Note also that errors at frequencies that are an even multiple of the spindle speed (such as 'synchronous' spindle errors) do not appear on the radial trace due to this aliasing.

The imparting of the tool geometry onto the work piece can be considered a convolution in the time domain. Conveniently, convolution in the time domain is equivalent to multiplication in the frequency domain. Therefore, the imparting of the tool geometry onto the work piece in the frequency domain can be considered a filter.

4. Validation through Experimentation

We are in the process of validating this procedure through actual machining tests. Our test bed is a T-based lathe. We will demonstrate this procedure for facing and cylindrical turning of copper using a diamond tool. For these simple cases, the dominant elemental errors include the feed axis straightness and the spindle axial, radial and tilt errors. During an actual error budgeting procedure, the form for the elemental error spectrums must be assumed based on previous experimentation and experience along with expert knowledge and analysis of the machine design. However, for this validation we will actually measure the frequency content of the elemental errors and then employ this procedure to predict the net spectrum of errors that will reside on our test parts. We will then machine these test parts and characterize the errors in the frequency domain using a combination of surface finish and form measuring instruments. The predicted spatial-frequency spectrum will be compared to the measured errors for final validation.

4.1 Results from Spindle Tests

The test machine has an air-bearing spindle with a DC brushless motor, pulse-width-modulated controller and a resolver for feedback. We measured axial and radial error motions of the spindle using a test mandrel with two precision balls and capacitance gages. Because we are interested in the spatial rather than temporal frequency content of the errors, we used an encoder to trigger data acquisition, gathering one thousand points every revolution of the spindle. We recorded twenty revolutions of data for spindle speeds from 60 RPM to 1500 RPM in steps of 60 RPM. As expected, the air-bearing spindle is very repeatable with sub-micrometer levels of asynchronous motion. However, the error characteristics drastically change at different spindle speeds. For example, the axial motion at 840 RPM spindle speed has a synchronous error with a dominant lobing of 17, 18 and 19 cycles per revolution as shown in

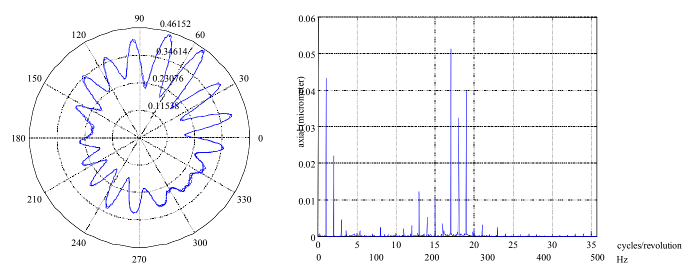

Figure 2: Axial Errors at 840 RPM

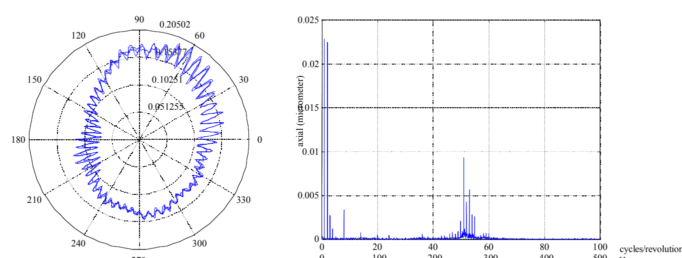

Figure 3: Axial Errors at 300 RPM

Figure 2. If this error was associated with a physical property of the motor such as the number of commutations, we would expect the spatial frequency of the lobing to remain fixed. However, at a spindle speed of 300 RPM, we observed a much higher spatial frequency lobing pattern as seen in Figure 3. In general, the spatial frequency of the lobing increases with decreasing spindle speed. However, the temporal frequency of the dominant errors remain in the same region for all spindle speeds as shown in the plots on the right sides of Figures 2 and 3.

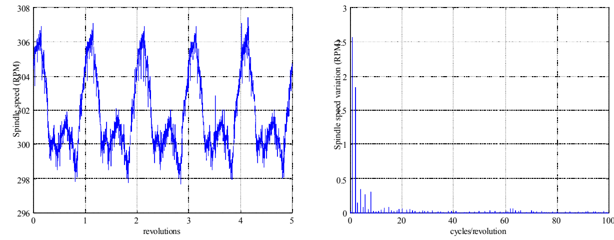

Figure 4: Spindle Speed Variation at 300 RPM

We are investigating the source of the forcing function. It is curious that the forcing function remains almost completely synchronous. Our hypothesis is that the forcing function is due the spindle speed variations about the set point caused by the controller. The observation that all but the once and twice per revolution errors disappear when the motor is turned off and the spindle coasts supports this hypothesis. Figure 4 shows the measured spindle speed when the desired speed is set to 300 RPM. Converting this variation to the spatial frequency domain, we observe that the variation is almost entirely synchronous. Modal testing revealed a torsional mode that excites axial displacements at approximately 240 Hz, which is the region where the errors are amplified. Therefore, we observe that the forcing function is amplified at the machine resonance, which is fixed in the temporal frequency domain. This corresponds to amplification at different spatial frequencies for different spindle speeds. Since we don't have similar data for other spindles, we are not sure whether these observations are unique to this machine or if this is a common occurrence. From a controls standpoint, it is very difficult to control the velocity of a system with almost no friction such as an air-bearing spindle. Furthermore, there is no velocity feedback sensor. We will continue to study the control system to determine the source of the spindle variations.

This work presents an opportunity to optimize cutting parameters to minimize errors. At low spindle speeds, the machine resonance results in a high spatial frequency error. However, since the forcing function does not have much energy at this high spatial frequency, the resulting errors are small. Unfortunately, slow spindle speeds cause longer machining times that could result in more significant thermal errors. At higher spindle speeds, the forcing function energy at the machine resonance increases, and we begin to see larger errors. However, it may be possible to select a high spindle speed that misses the machine resonance. During machining experiments for validation, we will select specific spindle speeds to study this effect.

5. Conclusions

This work points out the need for a mechatronic, or holistic approach to machine design. In other words, placing a precision air-bearing spindle on a machine is not enough to ensure small errors. We must consider the dynamic characteristics of the machine structure, control system and cutting parameters as well as the machine components and how they interact as a system to design a precision machine tool. This spatial-frequency-based error budget will encompass the entire machine design as an entire system, the end result will be the entire spatial frequency content of errors that would reside on the machined part. Work will continue for the next year, where we will complete measurements of contributing errors and determine the cutting process transfer function. We will then perform machining experiments and compare the measured errors to the prediction from this error budgeting procedure.

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